1. (10 points) Calculate the line integral,

$$\int_C \frac{-(y-1)}{(x-2)^2+(y-1)^2}\,dx \;+\; \frac{x-2}{(x-2)^2+(y-1)^2}dy$$

where

(a) C is the circle of radius 1 centered at $(2,1)$, oriented counterclockwise.

(b) C is the rectangle with vertices at $(1,0)$, $(3,0)$, $(3,2)$ and $(1,2)$ oriented counterclockwise.

2. (10 points) Calculate the line integral,

$$\int_C 2z\,dx + (2x+z)dy + (3x+2y)dz$$

where C is the curve of intersection of the cylinder $x^2+z^2=1$ with the plane $x+2y+z=1$ oriented counterclockwise when viewed from $(0,1,0)$.

3. (10 points) Use the method of Lagrange multipliers to find the point on the circle $x^2+(y-1)^2=1$ which is closest to $(2,0)$.

4. (10 points) Compute the outward flux of the vector field $\vec{F} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ across the surface $S = S_1 \cup S_2$, where S_1 is the portion of the upper sheet of a circular cone of aperture α inscribed into the unit sphere centered at $(0,0,0)$ and S_2 is the corresponding spherical cap, as in the following diagram

5. (10 points) Given the vector field

$$\vec{F} = (x^2 + y^2 + z^2)^{-3/2} \cdot (x, y, z) + \vec{\nabla} \times (\cos x,\ y^3 \tan xy, z).$$

Compute $\int\int_S \vec{F} \cdot d\vec{S}$, where S is the ellipsoid,

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1.$$

6. (10 points) Compute

$$\int\int\int_W z\, dxdydz,$$

where W is the tetrahedron with vertices:

$A(1, 6, 2),\ B(0, 1, 4),\ C(-1, 2, 3)$ and $D(0, 3, 1)$.

McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-265A

ADVANCED CALCULUS

Examiner: Professor N. Kamran
Associate Examiner: Professor J. Toth

Date: Tuesday, December 7, 1999
Time: 9:00 A.M. - 12:00 Noon

INSTRUCTIONS

Calculators are not permitted.

This exam comprises the cover and one page of questions.