McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-265B

ADVANCED CALCULUS

Examiner: Professor D. Jakobson
Associate Examiner: Professor J. Labute

Date: Thursday, April 18, 2002
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Faculty standard calculators are permitted.
This is a closed book examination.
Answer all questions.

D. J.

Dmitry Jakobson
John Labute

This exam comprises the cover and 1 page of questions.

189-265B, Advanced Calculus, Winter 2002.

FINAL EXAMINATION

Do all six problems.

Problem 1. Compute the integral $\int\int\int_W x\,dx\,dy\,dz$ where W is the tetrahedron with vertices at $(0,0,0),(0,1,2),(-1,4,3),(2,3,5)$.

Problem 2. Let

$$\overrightarrow{F} = -\text{grad}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right) + \text{curl}(e^{xy}, \cos(xz), x^2y^2z^3).$$

Compute the outward flux of $\overrightarrow{F}$ across the surface $x^2+y^2 = 6y+7-z^2$.

Problem 3. Using Stokes's Theorem, compute the integral $\int_C (z-y)dx+(x-z)dy+(y-x)dz$, where C is the curve $x=\sin^2 t, y = 2\sin t\cos t, z = \cos^2 t, 0 \leq t \leq \pi$. Hint: C is an ellipse lying in the plane $x+z=1$.

Problem 4. Prove that the vector field $\overrightarrow{F} = (yze^{xyz}, xze^{xyz}+4y^3z, xye^{xyz}+y^4)$ is conservative. Find the potential for $\overrightarrow{F}$ and evaluate the integral

$$\int_C \overrightarrow{F}\cdot d\overrightarrow{r},$$

where C is the straight line segment from $(-1,1,0)$ to $(2,-2,0)$, followed by the curve $x=2, y=-2\cos t, z = 2\sin t, 0\leq t\leq(\pi/2)$.

Problem 5. Find the area of the part of the surface of the torus parametrized by $x = (2+\cos\phi)\cos\theta, y = (2+\cos\phi)\sin\theta, z = \sin\phi$ where the parameters θ, ϕ lie in the region $0 \leq \theta \leq (2\pi)/3$ and $0\leq\phi\leq\pi/4$.

Problem 6. Using polar coordinates, compute the area of the region bounded by the curve $(x^2+y^2)^2 = 2(x^2-y^2)$ and lying *outside* the curve $x^2+y^2=1$. You may the following formulas useful: $\cos(2t) = \cos^2 t - \sin^2 t$, and $\sin(2t) = 2\sin t\cos t$.