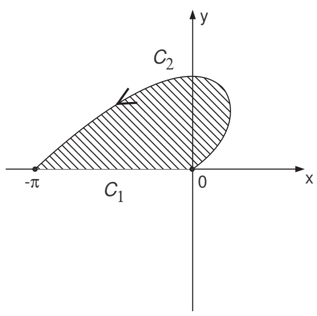
MATH 264B Advanced Calculus, Winter 2006

Assignment 4, due in class on Thursday March 16, 2006

1. Find the divergence and curl of the vector field

$$\mathbf{F}(x,y,z) = \frac{y}{\sqrt{z}}\,\mathbf{i} - \frac{x}{\sqrt{z}}\,\mathbf{j} + \sqrt{xy}\,\mathbf{k}.$$

2. Using Green's Theorem compute the area of the shaded region shown below, whose boundary consists of $\mathcal{C}_1 =$ the line segment $(-\pi, 0)$ on the x-axis together with the curve $\mathcal{C}_2 : \mathbf{r}(t) = t\cos t\,\mathbf{i} + 2t\sin t\,\mathbf{j},\ 0 \le t \le \pi$.



3. Using Green's Theorem compute the integral

$$\oint_{\mathcal{C}} (x+y)^2\,dx - (x^2+y^2)\,dy,$$

where $\mathcal{C}$ is the border of the triangle with vertices $(1,0)$, $(3,0)$ and $(3,2)$, oriented counterclockwise.

4. Using Green's Theorem compute the integral

$$\oint_{\mathcal{C}} (xy+y^2)\,dx + (x^2-y)\,dy,$$

where $\mathcal{C}$ is the border of the annulus D defined by $1 \le x^2+y^2 \le 9$ and $\mathcal{C}$ is positively oriented with respect to D.

5. Using Green's Theorem compute the circulation of the planar vector field

$$\mathbf{F}(x,y) = \frac{1}{2}(-y\mathbf{i} + x\mathbf{j}),$$

around the cardioid $r = 1+\cos\theta$ oriented counterclockwise.