MATH 264B Advanced Calculus, Winter 2006

Assignment 1, due in class on Thursday January 19, 2006, Solution **outlines**.

1. The integral to be computed reduces to

$$\int_0^3 \left[\int_{x=-y/2}^{x=y} x \sin y \, dx\right] dy = \frac{3}{8}\int_0^3 y^2 \sin y dy = \frac{3}{8}(-7\cos 3 + 6\sin 3 - 2)$$

2. Let $u = x + 2y$, $v = 3x - y$. By the change of variables formula for double integrals, we obtain

$$\iint_D \frac{x+2y}{3x-y} dxdy = \int_{-2}^1 \left[\int_1^3 \frac{u}{v} \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dv\right] du.$$

Now

$$\frac{\partial(u,v)}{\partial(x,y)} = -7,$$

so our integral reduces to

$$\frac{1}{7}\int_{-2}^1 u \, du \int_1^3 \frac{dv}{v} = -\frac{3}{14}\log 3.$$

3. The integral to be computed reduces to

$$\int_0^1 \left[\int_{y=0}^{y=1-x} \left[\int_{z=0}^{z=\frac{1}{3}(1-x-y)} zdz\right] dy\right] dx = \frac{1}{18}\int_0^1 \left[\int_0^{1-x} (1-x-y)^2 dy\right] dx = \frac{1}{216}.$$

4. By passing to polar coordinates, our integral reduces to

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left[\int_0^1 r^2 \cos^2 \theta r dr\right] d\theta = \frac{\pi}{8}.$$

5. Using cylindrical coordinates, our integral reduces to

$$\int_0^{2\pi} \left[\int_{r=0}^{r=1} \left[\int_{z=0}^{z=r} z \, dz\right] r \, dr\right] d\theta = \frac{1}{2}\int_0^{2\pi} \left[\int_0^1 r^3 \, dr\right] d\theta = \frac{\pi}{4}.$$