Department of Mathematics and Statistics, McGill University
MATH 265 ADVANCED CALCULUS: ASSIGNMENT 6

This assignment is due in class on Tuesday, April 6, 2004.

1. Evaluate the outward flux $\int\int_S \mathbf{F}\cdot\mathbf{N}dS$ of the following vector fields $\mathbf{F}(x,y,z)$ over the closed surface S formed by the hemisphere $z=\sqrt{a^2-x^2-y^2}$ and the plane $z=0$: (a) $\mathbf{F}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$; (b) $\mathbf{F}=x^2y\mathbf{i}+xy^2\mathbf{j}$; (c) $\mathbf{F}=xyz(\mathbf{i}+\mathbf{j}+\mathbf{k})$.

2. Verify Stokes theorem (by explicitly evaluating both the surface and the line integrals) for the surface $z=x^2+y^2$ with $z\le 4$ and the vector field $\mathbf{F}=y^2\mathbf{i}+x\mathbf{j}+z^2\mathbf{k}$.

3. Let $\mathcal{C}$ be the intersection of the cylinder $x^2+y^2=9$ with the plane $x+y+2z=1$ taken counterclockwise when viewed from above. Use Stokes theorem to evaluate $\oint_{\mathcal{C}}(2y+z)dx-(3x+z)dy+(3x-2y)dz$.

4. (Adams 16.5.10) Let $\mathcal{C}$ be the curve determined by the equations $(x-1)^2+4y^2=16$ and $2x+y+z=3$, oriented clockwise when viewed from high on the z-axis. Let

$$\mathbf{F}=(z^2+y^2+\sin x^2)\mathbf{i}+(2xy+z)\mathbf{j}+(xz+2yz)\mathbf{k}.$$

Evaluate $\oint_{\mathcal{C}}\mathbf{F}\cdot d\mathbf{r}$.

5. Let S be the portion of the paraboloid $z=9-x^2-y^2$ that lies above the plane $z=0$ and $\mathbf{N}$ denote the upward normal. Let $\mathbf{F}=(y-z)\mathbf{i}-(x+z)\mathbf{j}+(x+y)\mathbf{k}$. Calculate $\int\int_S \nabla\times\mathbf{F}\cdot d\mathbf{S}$ (recalling $d\mathbf{S}=\mathbf{N}\,dS$). .

6. Use Stokes' Theorem to evaluate the following, orienting the curves counter clockwise when seen from above:

   (a) $\oint_{\mathcal{C}} x\,dx+(x+y)\,dy+(x+y+z)\,dz$, where $\mathcal{C}$ is the curve of intersection of the cylinder $x^2+y^2=1$ and the plane $z=x$.

   (b) $\oint_{\mathcal{C}}(y+z)\,dx+(z+x)\,dy+(x+y)\,dz$ where $\mathcal{C}$ is the curve of intersection of the surfaces $x^2+y^2/2+z^2/3=1$ and $z=x^2+2y^2$.

   (c) $\oint_{\mathcal{C}} y^2\,dx+xy\,dy+xz\,dz$ where $\mathcal{C}$ is the curve of intersection of the cylinder $x^2+y^2=2ax$ and the plane $z=x$.

7. Suppose that a smooth vector field $\mathbf{F}(x,y,z)$ given on a region D has the property that on the bounding surface S it is perpendicular to the surface. Show that

$$\int\int\int_D \nabla\times\mathbf{F}\,dV=0.$$

March 19, 2004.